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CONDITIONS OF AERODYNAMIC INSTABILITY OF AIRFOIL EQUILIBRIUM POSITIONS

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Conditions of aerodynamic instability of airfoil equilibrium positions in an air flow are discussed.

In studying the conditions of losing the aerodynamic damping (as a result of aerodynamic stall) by an airfoil rotating about a fixed axis, Glauert [1] derived the following inequality as a necessary condition of autorotation:

$$C_y' + C_x < 0, \tag{1}$$

where C_y and C_x are the stationary aerodynamic lift and drag coefficients; the dash stands for the derivative with respect to the angle of attack α. The conclusions of [1] were verified on the experimental data published in [2].

Large vibrations (with amplitudes greater than a meter) of phase wires, mainly in the vertical direction along the OY axis (Fig. 1), were observed repeatedly in high-voltage power lines. In the special literature such motions are called a dance or galloping of a single wire of a power line.

Condition (1) was obtained by Den-Hartog [3] in studies of one-dimensional motion of a mechanical model of a wire as a necessary condition of self-excited vibrations in the vertical plane.

Thus, the consideration of one-dimensional self-excited vibrations of an airfoil led to condition (1), which may rightfully be called the Glauert–Den-Hartog condition.

It is interesting to note that, in our opinion, Russian researchers are not familiar with the paper of Glauert [1]. Thus, for example, there are no references to [1] in [4], although stability questions are considered, and other papers of Glauert are cited.

Fedyaevskii and Blyumina [5] studied the conditions of loss of aerodynamic damping by an angle airfoil rotating about a fixed axis. They obtained a rather complex inequality containing moment aerodynamic coefficients and came to the conclusion (from experimental data) that condition (1), which they call Den-Hartog's condition, can also be used as a criterion. Therefore, in our opinion, it would be useful for researchers to know that the book-stacks of the Russian State Public Library of Scientific and Technical Literature contain rather complete sets of the Proceedings of the British Advisory Committee for Aeronautics for 1909–1980.

Condition (1) was confirmed experimentally [5, 6] and has been applied in civil engineering. In designing skyscrapers, which are subjected to wind loads, the cross section of the building must be oriented so that the angles of attack with respect to the direction of prevailing winds are beyond the interval in which inequality (1) holds [7].

In simulating the behavior of a power-line wire under the action of wind, Van'ko [8] considered motion of an airfoil with three degrees of freedom: motion along the OX and OY axes and rotation about the center of mass (Fig. 1).

The conditions of existence of static solutions of the system of equations of motion were established: if the airfoil is lifting and the lift $Y \neq 0$, equilibrium positions exist for any wind (air flow) velocity $V > 0$.

In studies of the Lyapunov stability of possible equilibrium positions, the following results were obtained:

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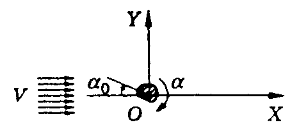

Fig. 1

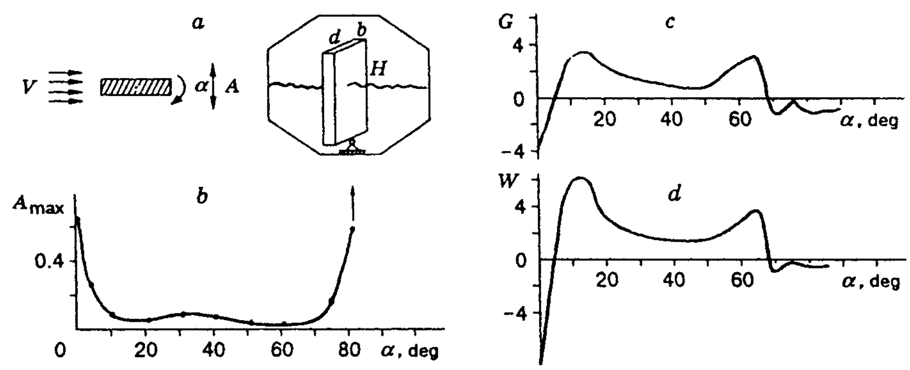

Fig. 2

— condition (1) is a sufficient condition of instability of equilibrium positions for a model with one degree of freedom (a special case of the model considered in [8]) if viscous resistance is ignored in the model;

— a sufficient condition of instability of equilibrium positions that includes only the aerodynamic characteristics of an airfoil (three degrees of freedom) was derived:

$$C_x(C_y' + C_x) + C_y(C_y - C_x') < 0. \qquad (2)$$

Upon division by $C_x^2 \neq 0$ condition (2) reduces to the form

$$K' + 1 + K^2 < 0, \qquad (3)$$

where $K = C_y/C_x$ is the lift-to-drag ratio (LDR) of an airfoil. For satisfaction of inequality (3), it is necessary that

$$K' < 0. \qquad (4)$$

As is known, if the derivative of LDR with respect to the angle of attack is negative, an aircraft in stationary flight can fall down into a spinning motion [9].

Thus, the instability condition derived in [8] is the condition of negative increments of the LDR with increasing angle of attack.

It is noted in [10] that for bluff airfoils at certain angles of attack the inequality $C_y - C_x' < 0$ is true. Therefore, the second term in inequality (2) is negative in the intervals of angle of attack in which $C_y > 0$, and the interval of angles for which the equilibrium of the model is unstable according to condition (2) is wider than that according to (1).

In the experiments on verification of the applicability of condition (1) to bluff airfoils, long prismatic rods of various cross-sections (rhombic, square, rectangular, and close to astroid) were supported on a cylindrical hinge and could vibrate only across the stream: typical galloping (Fig. 2a). Rigid rod models with elastic constraining linkages were tested in a T-1 wind tunnel with a closed test section at the Zhukovskii Central Aerohydrodynamic Institute; the tunnel cross-section is a regular octagon with an inscribed-circle diameter of 3 m; the free-stream turbulence intensity is independent of velocity and equal to 0.6%. Rigid horizontal tie-rods were attached to the model and linked to spiral strings which were outside the air flow. Varying string rigidity one can obtain a wide range of model vibration frequencies, i.e., kinematic Strouhal numbers Sh_k. The normal decrement of model vibrations is 0.03. The conditions of the experiments excluded

TABLE 1

α, deg	Airfoils				α, deg	Airfoils			
	Rhombic  $\alpha = 0$		Rectangular  $\alpha = 0$			Square  $\alpha = 0$		"Astroid"  $\alpha = 0$	
	C_x	C_y	C_x	C_y		C_x	C_y	C_x	C_y
0	0.7	0	0.66	0	0	1.4	0	1.67	0
5	0.65	0.07	0.54	−0.45	2.5	1.4	−0.01	1.65	0.06
10	0.64	0.15	0.56	−0.36	5	1.4	−0.24	1.63	0.09
15	0.62	0.27	0.6	−0.15	7.5	1.38	−0.38	1.6	0.11
20	0.6	0.41	0.66	0.08	10	1.36	−0.56	1.55	0.14
25	0.6	0.64	0.75	0.2	12.5	1.34	−0.74	1.5	0.17
30	0.75	0.41	0.85	0.32	15	1.2	−0.78	1.45	0.21
35	0.80	0.25	0.9	0.4	17.5	1.2	−0.6	1.39	0.26
40	0.82	0.05	1.0	0.46	20	1.2	−0.42	1.31	0.33
45	0.83	−0.15	1.06	0.5	22.5	1.25	−0.26	1.25	0.4
50	0.8	−0.35	1.1	0.54	25	1.32	−0.14	1.2	0.47
55	0.72	−0.26	1.12	0.56	27.5	1.38	−0.06	1.15	0.55
60	0.8	−0.1	1.14	0.6	30	1.4	0	1.12	0.63
65	0.9	0.03	1.1	0.72	32.5	1.44	0.04	1.08	0.75
70	0.98	0.1	1.2	0.7	35	1.48	0.06	1.02	0.8
75	1.05	0.13	1.3	0.5	37.5	1.49	0.08	1.0	0.4
80	1.1	0.13	1.4	0.4	40	1.5	0.07	0.98	0.2
85	1.13	0.08	1.42	0.19	42.5	1.5	0.04	0.96	0.1
90	1.14	0	1.44	0	45	1.52	0	0.97	0

wind resonance, since the aerodynamic Strouhal numbers were smaller than their kinematic values for each airfoil: $\mathrm{Sh_d} < \mathrm{Sh_k}$ [5]. Thus, for rectangular prisms, $\mathrm{Sh_k} \simeq 0.01$.

The largest vibration amplitude of the upper cross section versus the angle of attack $A = A(\alpha)$ was recorded for a rod of each particular shape at a fixed free-stream velocity.

The steady-state characteristics C_x and $C_y(\alpha)$ were determined in the same wind tunnel on a 4KAT-1 balance:

$$C_x = 2X/(\rho V^2 S), \qquad C_y = 2Y/(\rho V^2 S).$$

Here X and Y are the aerodynamic forces; ρ is the air density; and S is the lateral surface area.

The airfoils had sharp edges, which were the places of possible flow separation. For such bodies the separation points do not move, and, therefore, the steady-state aerodynamic characteristics are independent of the free-stream velocity. The coefficients C_x and C_y were determined at one and the same velocity that corresponded to $\mathrm{Re} = 0.5 \cdot 10^6$.

From the experimental curves $C_x = C_x(\alpha)$ and $C_y = C_y(\alpha)$ we calculated the functional relations

$$G(\alpha) = C_y' + C_x, \qquad W(\alpha) = K' + 1 + K^2. \tag{5}$$

The derivatives with respect to the angle of attack in (5) were calculated by second-order finite-difference formulas.

Table 1 presents the experimental data of the Industrial Aerodynamic Laboratory of the Zhukovskii Central Aerohydrodynamic Institute on measurement of steady-state values of the coefficients C_x and C_y for the above airfoils.

TABLE 2

α, deg	Airfoils: Rhombic $G(\alpha)$	Rhombic $W(\alpha)$	Rectangular $G(\alpha)$	Rectangular $W(\alpha)$	α, deg	Airfoils: Square $G(\alpha)$	Square $W(\alpha)$	"Astroid" $G(\alpha)$	"Astroid" $W(\alpha)$
0	1.50	2.23	−4.55	−8.55	0	−0.89	−0.60	3.04	1.83
5	1.57	2.46	−0.61	−0.04	2.5	−1.81	−1.29	2.32	1.43
10	2.02	3.36	2.97	5.92	5	−1.81	−1.49	2.09	1.31
15	2.22	4.03	3.24	5.32	7.5	−2.75	−1.90	2.29	1.50
20	3.24	5.86	2.04	2.68	10	−2.77	−2.04	2.24	1.54
25	−2.04	−3.82	2.12	2.33	12.5	0.42	−0.99	2.42	1.73
30	−1.08	−1.38	1.77	1.92	15	5.33	4.86	2.60	1.99
35	−1.49	−1.79	1.59	1.38	17.5	5.32	4.57	2.99	2.52
40	−1.47	−1.84	1.46	1.35	20	4.87	4.33	2.91	2.62
45	−1.46	−1.91	1.52	1.44	22.5	4.00	3.34	2.85	2.75
50	1.83	2.07	1.33	1.34	25	3.15	2.62	3.03	3.14
55	2.55	3.84	1.58	1.55	27.5	2.75	1.92	2.98	3.16
60	2.29	2.48	2.52	2.75	30	2.32	1.69	3.87	4.34
65	1.70	1.79	3.39	3.51	32.5	1.90	1.23	2.23	3.54
70	1.32	1.26	−1.09	−0.94	35	1.94	1.23	−8.15	−7.19
75	1.05	0.95	0.15	0.01	37.5	1.26	1.00	−3.58	−3.33
80	0.53	0.01	−1.01	−0.66	40	0.81	0.54	−1.31	−1.25
85	0.21	0.19	−0.76	−0.52	42.5	0.58	0.31	−1.33	−1.38

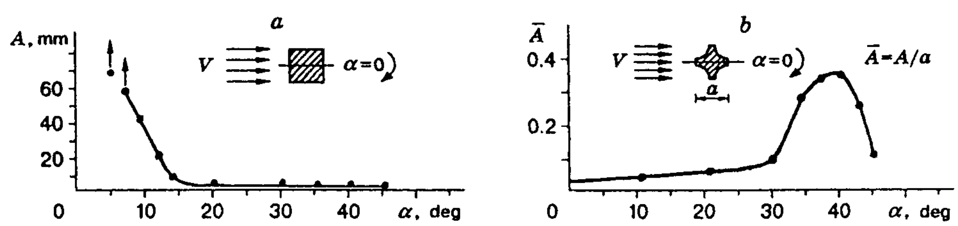

Fig. 3

Table 2 lists the results of processing of the data from Table 1 according to instability conditions (1) and (3); for convenience, the values satisfying inequalities (1) and (3) are underlined.

The curves of $A(\alpha)$, $G(\alpha)$, and $W(\alpha)$ are shown in Figs. 2b–2d for a rod of rectangular cross-section with height $H = 2$ m and section side ratio $b/d = 0.5$ ($b = 0.2$ m and $d = 0.4$ m).

The dependence of the largest amplitudes on the angle of attack for a rectangular cross-section shows that the loss of aerodynamic stability takes place in the vicinity of the zero incidence $0 < \alpha < 5°$ and for the angles of attack $70° < \alpha < 90°$ (Fig. 2b).

The instability zones determined from conditions (1) and (3) coincide with the experimental instability intervals (Figs. 2c and 2d). It should be noted that similar curves for a rhombic cross-section are presented in [11].

The experimental curves of $A = A(\alpha)$ for square and "astroid" cross-sections are shown in Figs. 3a and 3b. All data for the latter cross-section are borrowed from [5] and presented for completeness of the investigation of adequacy of instability conditions (1) and (3).

Thus, the described experimental data and computation results strongly support the adequacy of instability conditions (1) and (3). As was noted above, the instability interval determined from condition (2) can be wider than the corresponding interval calculated from (1) (see also Table 1 in [11]).

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